Supplemental material
to
"On the spontaneous magnetization of two-dimensional ferromagnets"

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I. DERIVATION OF EQUATION 1: MAGNETOSTATIC ENERGY OF JUXTAPOSED SLABS WITH OPPOSITE PERPENDICULAR MAGNETIZATION.

A. General considerations.

According to [1], the magnetostatic self-energy of a continuous distribution of permanent magnetization $\vec{M}(\vec{r})$ can be written as

$$E_M[\vec{M}(\vec{r})] = -\frac{\mu_0}{2}\cdot\iiint dV\cdot\vec{M}(\vec{r})\cdot\vec{H}(\vec{r}) - \frac{\mu_0}{2}\cdot\iiint dV\cdot\vec{M}^2(\mathbf{r})) \tag{1}$$

For the purpose of analyzing the situation of juxtaposed slabs with opposite perpendicular magnetization we use a sligthy different version for E_M. As $\vec{B} = \mu_0\cdot(\vec{H}+\vec{M})$ and $\iiint dV\cdot\vec{B}(\vec{r})\cdot\vec{H}(\vec{r}) = 0$ we can write

$$E_M = -\frac{1}{2\mu_0}\iiint dV\cdot\vec{B}(\vec{r})\cdot\vec{B}(\vec{r}) \tag{2}$$

With $\vec{B} = \vec{\nabla}\times\vec{A}$ and the partial integration

$$\iiint_V \vec{\nabla}\times\vec{A}\cdot\left(\nabla\times\vec{A}\right)dV = -\int_{\partial V}\left(\nabla\times\vec{A}\right)\times\vec{A}\cdot d\mathbf{S} + \iiint_V\left(\nabla\times\nabla\times\vec{A}\right)\cdot\vec{A}\,dV \tag{3}$$

we obtain, using $\vec{\nabla}\times\vec{B} = \mu_0\cdot\vec{\nabla}\times\vec{M}$,

$$E_M[\vec{M}(\vec{r})] = -\frac{1}{2\mu_0}\int dV\cdot\vec{A}\cdot\underbrace{\vec{\nabla}\times\vec{\nabla}\times\vec{A}}_{\mu_0\cdot\vec{\nabla}\times\vec{M}} = -\frac{1}{2}\int dV\cdot\vec{A}\cdot\vec{\nabla}\times\vec{M} \tag{4}$$

(the surface terms are rendered vanishing by extending the surface to ∞, where the fields of a finite and bounded magnetization distribution are vanishing). We recall that $\vec{A}$ fulfills the Poisson equation

$$\triangle\vec{A} = -\mu_0\cdot\vec{\nabla}\times\vec{M} \tag{5}$$

with solution

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\cdot\int dV'\frac{\vec{\nabla}\times\vec{M}}{|\vec{r}-\vec{r}'|} \tag{6}$$

Inserting this result in Eq.4 we obtain the sought-for representation of the total magnetostatic energy:

$$E_M[\vec{M}(\vec{r})] = -\frac{\mu_0}{8\pi}\iiint dV_1\iiint dV_2\frac{\vec{\nabla}_1\times\vec{M}(\vec{r}_1)\cdot\vec{\nabla}_2\times\vec{M}(\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|} \tag{7}$$

Eq.7 is the interaction energy of effective Amperian currents with current density vector

$$\vec{J}_M \doteq \vec{\nabla}\times\vec{M} \tag{8}$$

B. Application to perpendicular magnetization in a slab.

We compute explicitly the leading logarithmic contribution. Suppose we have two slabs with length L along the y-direction, infinitely extended along the the x-direction and with thickness $d << L$ along the z-direction. The slabs meet at $x = 0$. Along

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this line, the perpendicular magnetization changes from $(0,0,M_0)$ to $(0,0,-M_0)$. This magnetization jump produces an effective current density:

$$\vec{\nabla}\times\vec{M}=(0,\frac{2M_0}{w},0) \tag{9}$$

localized at $x=0$. This expression entails the fact that, for physical reasons (see Section III), the line $x=0$ at which the domains meet is assigned a finite width $w<<L$. A more accurate description of the current density would require the use of e.g. a tanh-profile for the domain wall, see Section III. In Eq.9, this profile is linearized, for simplicity.
The self-energy of the effective current flowing along the domain wall amounts to

$$-\frac{\mu_0}{8\pi}\cdot\frac{(2\cdot M_0)^2}{w^2}\iiint dV_1\iiint dV_2\frac{1}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}} \tag{10}$$

The integral along z extends from 0 to d, the integral along y from 0 to L and the integral along x from 0 to w. We use the variables $\vec{r}_1'=\frac{\vec{r}_1}{L}$, $\vec{r}_2'=\frac{\vec{r}_2}{L}$ and transform the integral to

$$-\frac{\mu_0}{8\pi}\cdot\frac{(2\cdot M_0)^2}{w^2}\cdot L^5\cdot\iiint dV_1'\iiint dV_2'\frac{1}{\sqrt{(x_1'-x_2')^2+(y_1'-y_2')^2+(z_1'-z_2')^2}} \tag{11}$$

The integral along z' extends from 0 to $\frac{d}{L}$, the integral along y' from 0 to 1 and the integral along x' from 0 to $\frac{w}{L}\doteq\omega$. We solve this integral in the limit $\frac{d}{L}<<1$. In this limit, we set $z_1'=z_2'=0$ in the integrand and perform the z'-integral to obtain

$$-\frac{\mu_0}{8\pi}\cdot\frac{(2\cdot M_0)^2}{w^2}\cdot L^5\cdot\frac{d^2}{L^2}\cdot\int_0^{\omega}dx_1'\int_0^{\omega}dx_2'\int_0^1 dy_1'\int_0^1 dy_2'\frac{1}{\sqrt{(x_1'-x_2')^2+(y_1'-y_2')^2}} \tag{12}$$

The remaining integrals are elementary ones and the result of the exact integration is

$$-\frac{\mu_0}{8\pi}\cdot\frac{(2\cdot M_0)^2}{w^2}\cdot L^3\cdot d^2\cdot 2\cdot\left\{\frac{\omega^2}{2}\ln\left(\frac{\sqrt{1+\omega^2}+1}{\sqrt{1+\omega^2}-1}\right)+\omega\ln\left(\sqrt{1+\omega^2}+\omega\right)-\frac{1}{3}\left[\left(1+\omega^2\right)^{\frac{3}{2}}-\omega^3-1\right]\right\} \tag{13}$$

We are interested in the situation where w is also much smaller than L while being larger than d. In this situation, Eq. 13 simplifies to

$$\approx-\frac{\mu_0}{8\pi}\cdot\frac{(2\cdot M_0)^2}{w^2}\cdot L^3\cdot d^2\cdot 2\cdot\left\{(\frac{w}{L})^2\ln\left(\frac{1}{\frac{w}{L}}\right)+(\frac{w}{L})^2\left(\ln 2+\frac{1}{2}\right)+\frac{(\frac{w}{L})^3}{3}+\mathcal{O}\left((\frac{w}{L})^4\right)\right\} \tag{14}$$

The leading term is the logarithmic one:

$$\approx-\frac{\mu_0}{4\pi}\cdot(2\cdot M_0\cdot)^2\cdot a^3\cdot\frac{L\cdot d^2}{a^3}\cdot\ln\frac{L}{w} \tag{15}$$

We use the parameter

$$\Omega\doteq\frac{\mu_0}{2}\cdot M_0^2\cdot a^3 \tag{16}$$

to write this leading term as (see Eq.1 in the bulk of the paper)

$$-\frac{2}{\pi}\cdot(\Omega\cdot\frac{d}{a})\cdot\frac{L\cdot d}{a^2}\cdot\ln\frac{L}{w}+\mathcal{O}(\frac{d^2\cdot L}{a^3}) \tag{17}$$

II. THE DIPOLAR CONTRIBUTION TO THE NEEL SURFACE ANISOTROPY: THE TERM $\Omega\cdot d$.

A. General considerations.

For the purpose of dealing with this specific problem we insert

$$\vec{H}=\frac{1}{4\pi}\cdot\iiint dV'\vec{\nabla}\frac{\vec{\nabla}'\cdot\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} \tag{18}$$

into

$$-\frac{\mu_0}{2}\cdot\iiint dV\cdot\vec{M}(\vec{r})\cdot\vec{H}(\vec{r}) \tag{19}$$

We use the identity

$$\vec{M}(\vec{r}')\cdot\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|}=\vec{\nabla}'\cdot\vec{M}(\vec{r}')\cdot\frac{1}{|\vec{r}-\vec{r}'|}-\vec{\nabla}'\cdot(\vec{M}(\vec{r}')\cdot\frac{1}{|\vec{r}-\vec{r}'|}) \tag{20}$$

and suppose that $M(\vec{r}')$ is well behaved and localized. Then the integral over the last term can be transformed by Gauss law into a surface integral and the surface can be pushed to infinity, where there is no magnetization and the surface integral vanishes, leading to

$$E_M[\vec{M}(\vec{r})]=+\frac{\mu_0}{8\pi}\iiint dV\iiint dV'\frac{\vec{\nabla}\cdot\vec{M}(\vec{r})\cdot\vec{\nabla}'\cdot\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|}-\frac{\mu_0}{2}\iiint dV\cdot\vec{M}^2(\vec{r}) \tag{21}$$

B. Application to the slab geometry.

We consider a slab of size $L \times L$ in the xy-plane, finite thickness $d << L$ along the z-direction. We need to compute the magnetostatic energy when the slab is filled with a uniform magnetization distribution $\vec{M} = (\sin\theta, 0, \cos\theta)$ (θ being the angle with respect to the slab normal), subtracted by the magnetostatic energy at $\theta = \frac{\pi}{2}$. As we will let the slab extend to infinity, we proceed by setting the derivative of $\vec{M}(\vec{r})$ along x and y to zero in Eq.21. The derivatives along z can be redirected and we obtain, for the magnetic anisotropy energy $\Delta E_M \doteq E_M[\cos\theta] - E_M[\cos\frac{\pi}{2}]$ produced by the dipolar interaction

$$\Delta E_M = \cos^2\theta \cdot \frac{\mu_0}{8\pi} \cdot M_0^2 \iiint d\vec{\rho} dz \iiint d\vec{\rho}' dz' \left[\frac{\partial}{\partial z} \frac{\partial}{\partial z'} \frac{1}{((\vec{\rho}-\vec{\rho}')^2 + (z-z')^2)^{1/2}} \right] \tag{22}$$

with $\vec{\rho} = (x, y)$. The integral over z and z' from 0 to d can be performed to obtain

$$\Delta E_M = \cos^2\theta \cdot \frac{\mu_0}{4\pi} \cdot M_0^2 \iint d\vec{\rho} \iint d\vec{\rho}' \left[\frac{1}{\sqrt{(\vec{\rho}-\vec{\rho}')^2}} - \frac{1}{\sqrt{(\vec{\rho}-\vec{\rho}')^2 + d^2}} \right] \tag{23}$$

The integrations over the in-plane coordinates are elementary, as the integration limits in the plane can be considered to extend to ∞ and $\vec{\rho}'$ can be set to zero:

$$\begin{aligned} \Delta E_M &= \cos^2\theta \cdot \frac{\mu_0}{4\pi} \cdot M_0^2 \underbrace{\int dx \cdot dy}_{L^2} \int dh \cdot dk \left[\frac{1}{\sqrt{h^2+k^2}} - \frac{1}{\sqrt{h^2+k^2+d^2}} \right] \\ &= \cos^2\theta \cdot \frac{\mu_0}{4\pi} \cdot M_0^2 \cdot L^2 \cdot 2\pi \cdot \underbrace{\lim_{L\to\infty} \int_0^L dr (1 - \frac{r}{\sqrt{r^2+d^2}})}_{d} \\ &= L^2 \cdot \frac{\mu_0}{2} \cdot M_0^2 \cdot d \cdot \cos^2\theta = (\frac{L}{a})^2 \cdot (\Omega \cdot \frac{d}{a}) \cdot \cos^2\theta \end{aligned} \tag{24}$$

In this last equation one can read out that $\Omega \cdot \frac{d}{a}$ determines the coefficient of the magnetic anisotropy arising from the dipolar interaction.

III. DERIVATION OF EQUATION 2 AND 3: THE WIDTH AND THE ENERGY OF A DOMAIN WALL.

This problem was solved originally by Landau and Lifschitz in 1935 [5]. Here we explicitly find the role of the dipolar interaction in determining the expression for the wall energy.

For the exchange energy between a pair of neighboring spins at sites $\vec{r}_i$ and $\vec{r}_j$ we adopt the classical rendering which appears to be appropriate e.g. for metallic Fe, as explained in Ref.[2]:

$$E_J = -J \cdot S^2 \cdot \vec{n}(\vec{r}_i) \cdot \vec{n}(\vec{r}_j) \tag{25}$$

$\vec{n}$ is a classical vector. $J \cdot S^2$ is the exchange coupling constant given in Ref.[2] as ≈ 46 meV. The use of S^2 instead of $S(S+1)$ is explained in Ref.[2]. For using this expression to computing the energy of a domain wall we need some approximations. Within a wall that evolves along the y-axis, $\vec{r}_i \doteq (0, y, 0)$ and the $\vec{r}_j \doteq (0, y \pm a, 0)$ and

$$\vec{n}_i = (\sin\theta(y), 0, \cos(\theta(y)) \quad \vec{n}_j = (\sin\theta(y\pm a), 0, \cos(\theta(y\pm a)) \tag{26}$$

The change of exchange energy produced by the misalignment of two consecutive spins amounts accordingly to

$$E_J(\uparrow\downarrow) - E_J(\uparrow\uparrow) = -J \cdot S^2 \cdot \cos(\theta(y) - \theta(y\pm a)) + J \cdot S^2 \tag{27}$$

We expect that $\theta(y) - \theta(y \pm a)$ is small, so that we can replace the cos function with the lowest terms of its Taylor series and obtain

$$E_J(\uparrow\downarrow) - E_J(\uparrow\uparrow) \approx \frac{JS^2}{2} (\theta(y) - \theta(y\pm a))^2 \tag{28}$$

We also use

$$(\theta(y) - \theta(y\pm a))^2 \approx \frac{\partial\theta(y)}{\partial y}^2 \cdot a^2 \tag{29}$$

This approximation allow to write the exchange component of the total elastic functional that is used to compute the equilibrium profile of $\theta(y)$. Together with the term originating from the Neel [3] magnetic anisotroy, the functional $E_w[\theta(y)]$ describing the total energy of a wall writes

$$E_w[\theta(y)] = \frac{L \cdot d}{a^2} \cdot \left[\frac{J \cdot S^2}{2} \cdot a \cdot \int_{-\infty}^{\infty} \left(\frac{\partial\theta(y)}{\partial y} \right)^2 \cdot dy - (\lambda - \Omega \cdot \frac{d}{a}) \frac{1}{a} \int dy \cdot \cos^2\theta(y) \right] \tag{30}$$

The corresponding Euler-Lagrange equation reads

$$J \cdot a \frac{d^2\theta}{dy^2} - 2\frac{\lambda - \Omega\frac{d}{a}}{a} \sin(\theta(x))\cos(\theta(y)) = 0 \tag{31}$$

The solution to the boundary conditions $\lim_{y\to-\infty}\theta(y) = \pi$ and $\lim_{y\to+\infty}\theta(y) = 0$ reads

$$\cos(\theta(y)) = \tanh\left(\frac{y}{w}\right) \tag{32}$$

with the width w of the wall

$$w \perp = \frac{a}{2}\sqrt{\frac{2 \cdot J \cdot S^2}{\lambda - \Omega\frac{d}{a}}} \tag{33}$$

(Eq. 2 in the bulk of the paper.) The total energy of the wall amounts to (see Eq.3 in the bulk of the paper)

$$\frac{L \cdot d}{a^2} \cdot 2 \cdot \sqrt{\left(\lambda - \Omega\frac{d}{a}\right) \cdot 2 \cdot J \cdot S^2} \doteq E_w \tag{34}$$

IV. STABILITY OF A STRIPE DOMAIN IN AN APPLIED MAGNETIC FIELD.

We have determined that, when the size of a magnetic element exceeds the crossover length L_c, a phase with domains of opposite perpendicular magnetization can penetrate a magnetic element. We now determine the stability of this phase with respect to a magnetic field applied perpendicularly to the plane. For this purpose, we compute the magnetic field necessary to render metastable one stripe of reversed magnetization with width δ. The change in total energy produced by the stripe with respect to an element of uniform magnetization $+M_0$ in an applied magnetic field $\vec{B} = (0,0,+B_0)$ amounts to

$$\Delta E(B_0,\delta) = 2 \cdot E_w \cdot \frac{L \cdot d}{a^2} + 2 \cdot B_0 \cdot M_0 \cdot \delta \cdot L \cdot d - \frac{4}{\pi} \cdot (\Omega \cdot \frac{d}{a}) \cdot \frac{L \cdot d}{a^2} \cdot \ln\frac{\delta}{w} + \mathcal{O}(\frac{L \cdot d}{a^2} \cdot \Omega \cdot \frac{d}{a}) \tag{35}$$

The $-\ln\frac{\delta}{w}$ in Eq. 35 comes about by subtracting $2 \cdot \ln\frac{L}{w}$ (originating from the the self energy of the effective current densities flowing along the domain walls) from $2 \cdot \ln\frac{L}{\delta}$ (originating from the reciprocal interaction energy between the effective current densities flowing along the domain walls [6]). Minimizing $\Delta E(B_0,\delta)$ with respect to δ produces the equilibrium stripe width

$$\delta^*(B_0) = a \cdot \frac{\frac{2}{\pi} \cdot \Omega \cdot \frac{d}{a}}{B_0 \cdot M_0 \cdot a^3} = d \cdot \frac{\mu_0}{\pi} \cdot \frac{M_0}{B_0} \tag{36}$$

On p.20-21 of Ref.[7], a numerical study of $\Delta E(B_0,\delta)$ as a function of δ for various fields B_0 is reported. It is shown that, below a certain threshold magnetic field B_t (and, in particular, at $B_0 = 0$), this minimum renders $\Delta E(B_0,\delta)$ negative: the energy of the element containing the stripe is lower than the energy of the element with uniform magnetization. Above the threshold field, instead, the minimum provides a metastable state, as $\Delta E(B_0,\delta^*) > 0$ [7]. We now proceed to find B_t. To find B_t we insert the expression for $\delta^*(B_0)$ in $\Delta E(B_0,\delta)$, i.e. we build the function $\Delta E(B_0,\delta^*(B_0))$:

$$\Delta E(B_0,\delta^*(B_0)) = \frac{L \cdot d}{a^2} \cdot \left(2 \cdot E_w - \frac{4}{\pi} \cdot (\Omega \cdot \frac{d}{a}) \cdot \ln\left(\frac{\mu_0 \cdot M_0}{\pi \cdot B_0} \cdot \frac{d}{w}\right) + \mathcal{O}(\Omega \cdot \frac{d}{a})\right) \tag{37}$$

$\Delta E(B_0,\delta^*(B_0))$ expresses the energy change at the minimum δ^* as a function of B_0. Setting $\Delta E(B_0,\delta^*(B_0))$ to zero provides us with an equation in the variable B_0 for the sought-for field B_t. The solution of this equation writes

$$B_t \propto \mu_0 \cdot M_0 \cdot \frac{d}{L_c} \tag{38}$$

This is the transition field referred to in the bulk of the paper. A similar dependence of B_t on M_0 and on the ratio L_c/d was obtained in a situation of stripe order in 2D[7, 8]. Notice that, even if the stripe is metastable, there is an energy barrier for it to be eliminated from the element. A further issue is that a bubble phase might intervene between the stripe phase and the phase of uniform magnetization. For the purpose of this paper, however, these issues are less relevant and we refer the reader to Refs.7 and 8 for an extended discussion.

V. ABOUT EQUATION 5: MAGNETOSTATIC ENERGY OF JUXTAPOSED DOMAINS WITH OPPOSITE IN-PLANE MAGNETIZATION.

For the in-plane configuration we compute the magnetostatic energy using

$$E_M[\vec{M}(\vec{r})] = +\frac{\mu_0}{8\pi}\int dV \int dV' \frac{\vec{\nabla} \cdot \vec{M}(\vec{r}) \cdot \vec{\nabla}' \cdot \vec{M}(\vec{r}')}{\mid \vec{r} - \vec{r}' \mid} \tag{39}$$

(the summand on the right hand side of the exact Eq.21 cancels out when energies of two different configurations with equal $\iiint dV \vec{M}^2(\vec{x})$ are subtracted). We consider the two magnetization distributions "uu" (a in the top view of the slab in Fig.1SM) and "ud" (b in Fig.1SM) (for a magnetization pointing along $+x$ and $-x$ we use the symbols "u" and "d", respectively). In uu the in-plane magnetization $\vec{M} = (0, M_0, 0)$ is uniformly distributed. It produces a $\vec{\nabla} \cdot \vec{M}$ in the vicinity of the segments terminating the slabs: effective negative charges accumulate along the segments A and B and effective positive charges along the segments C and D, see Fig.1SM. In *ud*, half of the slab is filled with $\vec{M} = (0, +M_0, 0)$ and the remaining half with the opposite magnetization $(0, -M_0, 0)$. The effective charges at the terminating segments are accordingly modified: A and D are negatively charged, B and C positively. In a situation where the domain boundary is exactly parallel to the magnetization vector, the self-energy of the charge distributions cancel out when the magnetostatic energy of the configuration *uu* is subtracted from the energy of the configuration *ud*. The remaining terms amount to the Coulomb interaction (symbolically) $-4 \cdot A^{(+)} B^{(+)} + 4 \cdot A^{(+)} D^{(+)}$. The leading term is the Coulomb energy between the charges along the segments A and B, and it is negative, i.e. the formation of domains with parallel opposite magnetization lowers the magnetostatic energy of the slab. In the evaluation of this energy, the integration along z produces a d^2 by simultaneously setting $z = z' = 0$ in the integrand. The integration along y produces Δ^2 by simultaneously setting $y = y' = 0$ in the integrand. Δ is the length of that border region at A, B, C, D across which the magnetization decays to 0. The integration over x and x' must be performed explicitly. We insert a wall of finite thickness w between the two domains. At the end of the calculation we will let w go to zero. This will show that this energy contribution is also finite. $E(ud) - E(uu)$ writes, approximately

$$\begin{aligned} E(ud) - E(uu) &\approx (-4) \cdot \frac{\mu_0}{8\pi} M_0^2 \cdot d^2 \cdot \int_{-L}^{-w} dx' \int_{w}^{L} dx \frac{1}{|x - x'|} \\ &= (-4) \cdot \frac{\mu_0}{8\pi} M_0^2 \cdot d^2 \cdot L \int_{-1}^{-\frac{w}{L}} dx' \int_{\frac{w}{L}}^{1} dx \frac{1}{|x - x'|} \\ &= (-4) \cdot \frac{\mu_0}{8\pi} M_0^2 \cdot d^2 \cdot L \int_{-1}^{-\frac{w}{L}} dx' \ln \frac{1 - x'}{\frac{w}{L} - x'} \\ &\underbrace{\approx}_{w \to 0} -4 \cdot 2 \cdot \ln 2 \cdot \frac{\mu_0}{8\pi} M_0^2 \cdot d^2 \cdot L = -\frac{2 \ln 2}{\pi} \cdot \Omega \cdot \frac{d^2 \cdot L}{a^3} \end{aligned} \tag{40}$$

This result is used to write Eq.5 in the bulk of the paper.

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[1] J.D. Jackson, Classical Electrodynamics, John Wiles and Sons, Inc., New York, Third Edition, p.230

[2] L. M. Small and V Heine, A couple method for calculating interatomic interactions in itinerant electron magnetic systems, *J. Phys. F: Met. Phys.* **14**, 3041-3052 (1984)

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[5] L. Landau and E. Lifshitz, On the theory of the dispersion of magnetic permeability in ferromagnetic bodies, Phys. Z. Sowjet. **8**, 153 (1935).

[6] To obtain this result we use

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{\sqrt{y^2 + \delta^2}} \cdot dy \approx 2 \cdot \ln \frac{L}{\delta} \tag{41}$$

This integral appears in the computation of the interaction between two Amperian current densities aligned along the x-direction, with length L much larger than their distance δ.

[7] N. Saratz, Inverse Symmetry Breaking in Low Dimensional Systems, Diss. ETH Zurich Nr. 18491, https://doi.org/10.3929/ethz-a-006007536, Logos Verlag, ISBN-13: 978-3832524036

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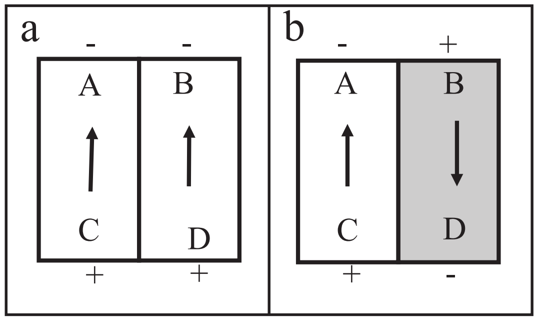

FIG. 1. a: Top view of the slab in the state of uniform in-plane magnetization (represented in white). Black arrows represent the magnetization vector. The sign of the effective charges appearing along the segments A,B,C,D is given. b. Top view of the slab with two domains of opposite in plane magnetization (white and gray), magnetization vectors represented by black arrows. The sign of the effective charges appearing along the segments A,B,C,D is given.